529. b. 10
3

ENNEADES ARITHMETICÆ;

THE

NUMBRING NINES.

OR,

PYTHAGORAS His TABLE

Extended to

All Whole Numbers under 10000.

AND

The Numbring RODS

Of the Right Honourable

JOHN. LORD NEPEER,

Enlarged

With 9999 Fixt Columns or Rods, of Single, Double, Triple and Quadruple Figures, and With a New Sort of Double and Moveable Rods, for the much more sure, plain and Easie performance of *Multiplication, Division*, and *Extraction of Roots*.

The Whole being very Useful for most Persons, of whatsoever Calling and Employment, in all Arts and Sciences.

All having frequent Occasions of *Accompts, Numbring, Measuring, Surveying, Gauging, Weighing, Demonstrating, &c.* The Devine Wisdom having from the Beginning

Disposed all things in Measure, Number and Weight, Sap. 11. 21

LONDON.

Printed for *Joseph Moxon*, at the Sign of *Atlas* in *Ludgate-street*. Where also these Numbring Rods, (commonly Call'd *Napiers Bones*) are made and Sold. 1684.

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To the READER.

Courteous Reader,

THe *End and Use of the Ensuing Table will be the better understood, if something in Brief, by way of Preface, be premised concerning these Three Points. First, the* Table of Pythagoras; *Secondly,* the Extensions of the same; *and Thirdly,* the Numbring Rods. *Wherefore be pleased to peruse the following Preamble concerning the said Points; The Reading whereof will not Cost thee much above an Hour or Two. But First, vouchsafe to hear, what is meant by the New Title of* Enneades Arithmeticæ, *or* Numbring Nines.

The whole following Table containing Gradually all the whole Numbers *from* 1. *to* 9999. *Inclusive, Viz.* 9. *of One single Figure,* 90 *of Two Figures;* 900 *of Three Figures; And* 9000 *of four Figures: Again, every one of the said whole* Numbers (*multiplied severally by all the* 9 *Unites,* 1, 2, 3, 4, 5, 6, 7, 8, 9.) *making so many* Columns, *as there are* Capital Numbers, *to wit,* 9999. *each Column, con-*

ſiſting of 9 diſtinct Numbers, and theſe 9 Numbers being the products of 9 diſtinct Units, I think we may with good Reaſon call the ſaid Columns (yea and the Numbring Rods alſo; For in effect they are the ſame thing) by the Name of Enneades Arithmeticæ, that is, the numbring Nines, or more expreſſively, the numbring Ninities. For the Greek word Ἐννεὰς being a Nown Subſtantive, ſignifies properly the Number of nine in abſtracto, which may as well be called a Ninitie, as Ἑνὰς ſignifying the Number of One in abſtracto, is interpreted, Unity, and Τριὰς ſignifying the Number of Three in abſtracto, is Tranſlated Trinity. But though there be but Nine Cells expreſt in every Rod, yet one other Negative Cell of Cyphers is ever to be underſtood, which if you pleaſe, may be ſufficiently inſinuated by putting ſo many Points over the Vertical Cell, as there are Figures in the Vertical; ſee Figure 10. whoſe Vertical is $\dot{1}\ \dot{2}\ \dot{3}\ \dot{4}$ and may be pointed, as you ſee here, to inſinuate a Suppoſed Cell of Cyphers.

Concerning

Concerning the Table of PYTHAGORAS.

THE known Arithmetical Table, invented by *Pythagoras*, (such as you see expressed in Figure 1.) is not only an easie and sure Rule to *multiply* and *divide* by, but is also those very Operations themselves, *Multiplication and Division*, done to your Hands, and known by inspection, comprehending three distinct Numbers, proper to them both, viz. *Multiplicand*, *Multiplier* and *Product*, proper to Multiplication, *Dividend*, *Divisor* and *Quotient*, proper to Division. For if you take any one of the Numbers, Seated in their several Cells between A and B. for a multiplicand; for example 8. and another Number of those that are Seated in their several Cells between A and C for a *multiplier*, for example 7. in the Angle of their Concurse. you will find the Number 56. the just Product of 8 *multiplyed* by 7. Again, the said *Product* 56. is also a *Dividend*, whose *Divisor* is 8 in the highest Cell, above the *Dividend* and *Quotient* is 7 in the 7th. Lateral Cell, over against the *Dividend*, the better to distinguish the 9 Units, Figures, Numbers and Cells Seated between A and B. from the like Seated between A and C. call the first Capital Units, Figures, Numbers, Cells, as

being placed in the Head of the Table: but the two Lying between A and C call *Lateral*, as occupying the side of the Table on the left Hand.

Every Capital number in the *Pythagorean* Table hath under it 8 other numbers lodged in 8 several quadrats or Cells, as you may see in Figure 1. all which 9 numbers make a kind of a little streight Column, parallel to the side A C or B D, The Columns are 9 answerable to the 9 *Units* or *Capital Numbers* in the Head of the Table. But observe also, that there are other 9 *Transverse Columns*, parallel to the side A B. or C D. which cross the former at Right Angles, and meet one another in a common Cell, ever containing a perfect *Quadrat Number*, whose Root appears in the Heads of the two Meeting Columns: For Example, the *Column* of 8 Capital meets with the *Transverse Col.* of 8 Lateral in the Cell of 64. a square Number. So 9 Capital Meets with 9 Lateral in the Cell of 81. a square Number, *&c.* But what is worthy of Observation, these two different sorts of Columns, *Capital and Transverse*, though most cross one to another, do most punctually agree in all their Numbers, without any difference, as is manifest to the Eye.

There are yet many things more, very Observable in the *Pythagorean* Table. The first is, that not only the 9 Units, are *multiplicand and Divisors* in it, but Tens, Hundreds, Thousands, 10000s, 100000. 1000000. 10000000 and 100000000. in

great variety, and all actually and orderly Tabulated, ſhewing at the ſame time, their true Product and Dividends, with *multiplyers*, *Diviſors*, and *Quotients*. As for *Tens* you ſee 12 Tabuated on the 1. and 2 *Col.* Then 23. 34. 45. 56 67. 78. 89. each number Tabulated on two Contiguous Columns. *As for Hundreds*, you ſee 123. Tabulated on the firſt 3 *Col.* Then 234 345. 456. *&c.* As *for Thouſands*, you have 1234. Tabulated on the four firſt *Col.* Then 2345. 3456. 4567. *&c.* and ſo of the reſt, till you come to 123456789. a *multiplicand* or *Diviſor* of all the *Capital Units*, of the Table, whoſe *multiplyer* is one (or more, as you pleaſe) of the Lateral Units, and the *Product* is the Tranſverſe Column of that *Unit*, which you chooſe for *multiplyer* to be counted from the Right Hand to the left. For example, if you *multiply* 123456789 by 2. the product will be the ſecond tranſverſe col. gathered from the right hand to the left, *viz.* 246913578. If you multiply it by 9. the product will be the 9th. tranſverſe column, *viz.* 1111111101.

The ſecond thing very obſervable is, that if you turn *Pythagoras* his Table in ſuch manner, that all the Numbers remain unchang'd in their cells, and yet the Figures 9. 8. 7. 6. 5. 4 3. 2. 1. lying between C. and A. become *Vertical*, which

before.

before were *Lateral*, and 1. 2. 3. 4. 5. 6. 7. 8. 9. lying between A. and B. become *Lateral*, which before were *Vertical*, you may find another great Variety of *Multipliers*, *Divisors*, *Products*, *Dividends* and *Quotients*, and of greater Numbers than before, all differing from the former, and all Tabulated on contiguous columns. As for tens, you see 98 Tabulated on the two last transverse columns, then 87. 76. 65 &c. As for Hundreds, you have 987 876. 765. &c. And so for the rest, till you come to 987654321 a Multiplicand or Divisor made of all the Lateral unites from C. to A. which Number multiplied by 2. will have for product 1975308642. to be found in the *second Capital Column*, and gathered thence from the right Hand to the left. If you multiply it by 9. the *capital Column* of 9 will shew the product 8888888889.

The third observable thing is, That whatsoever under-cell of any Column hath more figures or places in it, than are in the capital cell of that Column, then infallibly the Figure which is outmost on the left side of that under-cel; is to be added to the next Figure of another Column, if another Column be Tabulated by it on the left Hand. This Addition may be called *Collateral*, because it adds together two Figures on the sides of two Neigbouring Columns, and makes but one Number of them both. If the two Figures added

ded should make 10. then put down a cypher, and carry one to the next Number on the left Hand: If they make more then 10, put down the surplus and carry one. Take this example of *Collateral* Addition. If you Tabulate 12. with two Rods or Columns, *viz*. the column of 1 and the column of 2. in the 2d. Cell of both Rods together, is 24. in the 3d. Cell is 36. in the 4th. Cell is 48. but in the 5th. Cell is 510. which make not five Hundred and 10. but 60 only, because 1 and 5 (the Neighbouring Figures of 2 Columns) are to be added into one number 6, by reason that the 5th. Cell of the Column 2 hath 10. in it, a Figure more than in the Capital of the Column two. This Rule then is Universal, whatsoever under-cell of any column hath more Figures in it than are in the capital number of that column, there must be collateral Addition, if any other column be Tabulated on the Left Hand with it. Note that this Rule holds good, not only in columns of single Units, but of Tens, 100ds. 1000ds. &c.

The 4th. Observable thing is, and of cheif moment, that all and every column, *Ennead*, or Rod (Synomical words in the present matter) not only of *Pythag*. his Table, but of all following Tables to 9999 and much more, is singularly useful both in *Division* and *multiplication* though the Column be never so little (except the Column of 1 the first Unit, which in Rigour neither devides nor multiplys any Number) and the *Dividend* and *multiplyer*

never ſo great. For in Diviſion it performes the work, or gives the *Quotient*, by meer Subſtraction of its own Numbers out of the *Dividend*: and in *multiplication* it gives the Product, by ſetting down in due order its own numbers, and afterwards adding them into one Sum. For example, take the Column 25 and divide by it 7 8 9 6 5 2 5 the Quotient will be 3 1 5 8 6 1. and the work ended will appear as underneath (*a*) where note that the Numbers 75. 25. 125. 200. 150 25. all marked with this mark - are taken out of the cells of the column 25. to be ſubſtracted out of the *partial Dividends*: 75. is taken out of the third cell, and gives you 3. to be ſet in the *Quotient*; 25 is taken out of the 1 cell, and gives one for the *Quotient*, and ſo of the reſt, the Number ſhewing its cell, and the cell the *Quotient*. A-

```
(a) 25) 7896525 (315861.
        -75
          39
         -25
          146
         -125
           215
          -200
            152
           -150
              25
             -25
              00
```

gain;

gain, take the column 25. for a *Multiplicand.* and multiply it by 315861. the *product* will be 7896525, which before was the Dividend. The Operation ended, will appear, as underneath at (*b*), Where note, that the Numbers 25. 150.200. 125.25.and 75. are all taken out of the cells of the column 25. to be placed as you ſee, and added into one ſum for product of Multiplication. Here alſo you may obſerve, that the ſelf-ſame cells or Numbers are added together in *Multiplication*, which were ſubſtracted in *Diviſion*, only their Order inverted: what was firſt ſubſtracted in *Diviſion*, is laſt taken and added in *Multiplication*, which always happens when the *Diviſor* and *Quotient* become *Multiplicand* and *Multiplier* and reproduce the *Dividend* as *Product* of *Multiplication*.

The fifth thing obſerveable is, That every *Ennead* or column, be it never ſo little or great,

25
315861
(*b*) 25
150——5
200——2
125——5
25——6
75——9
7——8

that

that is, of one, or more, or many Figures in its *Capital Cell*) by multiplying its Capital Number with 45, will produce a sum equal to all the Figures, as they stand in that col. added into one sum. For example, take the col. of 6. and multiply 61. by 45. the product will be 2745. which is just the sum of all the column added together, as appears in the margent (c). By this means you may examine any col. whether it be right or wrong.

(c)	61
	122
	183
	244
	305
	366
	427
	488
	549
	2745

Add unto the former or fifth Observation another, not much unlike, to be seen in this little Table of *Roots*, *squares* and *cubes*, or rather of the ending Figures or Units of all Roots, Squares and Cubes whatsoever; where you see the sum of each column by Addition, to be severally 45. The *first Column* is of the first ten Roots, from 0 to 9 inclusive, but all following Roots have the same ending Figures, and in the same order, as in the first column. *The second Column* is of the first ten ending Figures of Squares; the first ten Squares, and all the following Squares have the

R.	sq	cub.
0	0	0
1	1	1
2	4	8
3	9	7
4	6	4
5	5	5
6	6	6
7	9	3
8	4	2
9	1	9
45	45	45

the ſame ending Figures, and in the ſame Order as in the ſecond column. *The third Column* is of the firſt ten ending Figures of the firſt ten Cubes, and all the following Cubes have the ſame ending Figures, and in the ſame order as in the third column. By the ending Figure of any Root, you may know the ending figure both of the ſquare and cube by this Table: in which the *ſquare* and *cube* ſtand right over againſt the *Roots*. Hence may you know, whether a Table of Roots, ſquares and cubes be well made or no: for if any ten ending *Roots*, or *Squares*, or *Cubes* lying next one under another do not make the ſum 45, or that the ſquares and cubes do not anſwer the roots, as in this Table, there muſt neceſſarily be an Errour committed.

The 6th. thing very remarkable, and indeed admirable is, that *multiplication and Diviſion* being two very diſtinct and different Operations, yet they ſo inſeparably and eſſentially accompany one another, that the one, for example, *Multiplication* can never be wrought or Finiſhed by its proper Rules, but that *Diviſion* at the ſame time ſhall be given you without working by any Rules of Diviſion: yea, when the Operator did neither intend Diviſion, nor ſo much as think of it. That they are two different Operations, it is clear. For

1. *Multiplication*, by two Numbers given (*multiplicand and multiplier*) ſeeks a third, *Viz.* the *Factum or Product*: But *Diviſion* by

two Numbers given, different from thoſe of *Multiplication*, (*Diviſor and Dividend*) ſeeks a third, *viz.* the *Quotient*, different from the product of multiplication.

2. *Multiplication* begins its work with the leaſt figure, and Ends it with the greateſt: but *Diviſion* quite contrary, begins with the greateſt and Ends with the leaſt.

3. *Multiplication* requires *Addition* only, without *Subſtraction*: But *Diviſion* requires *Subſtraction* only, without *Addition*. Notwithſtanding theſe differences of the two Operations, it is impoſſible to work a *Multiplication* but a *Diviſion* will be at the ſame inſtant given you, without working or dividing. So is it alſo impoſſible to work a *Diviſion* but a *Multiplication* ſhall be given you without working or multiplying. And the reaſon is manifeſt, becauſe the ſelf ſame three Numbers which conſtitute the Eſſence of *Multiplication* conſtitute alſo the Eſſence of *Diviſion*, though under different denominations. The three Numbers in Multiplication are called *Multiplicand*, *Multiplyer* and *Product*. In *Diviſion*, *Diviſor*, *Quotient* and *Dividend*. And obſerve, that by how much a *Multiplcand* exceeds or comes ſhort of his *Multiplyer*, by ſo much the *Diviſor* will exceed or come ſhort of his *Quotient*. The *Product of Multiplication* is ever equal to the *Dividend in Diviſion*. See the following example.

Multi-

Multiplication wrought		*Division Given*	
Multiplicand	144	*Divisor*	144
Multiplier	12	*Quotient*	12
Product	1728	*Dividend*	2728
Division wrought		*Multiplication Given*	
Divisor	7324	*Multiplicand*	7324
Dividend	4789896	*Product*	4789896
Quotient	654	*Multiplier*	654

Observe, that when in *Multiplication* the less Number is made the *Multiplicand*, and the greater the *Multiplier*; Then in *Division* given, the *Divisor* is the less Number, and the *quotient* the greater. Example.

Multiplicand	12	*Divisor*	12
Multiplier	144	*Quotient*	144
Product	1728	*Dividend*	1728

The 7th thing observable is, That the third, fourth and fifth *Cell* of every *Ennead* (whether it hath one, or more or many Figures in its Verticall, and those either pure integers or mixt with Fractions) contain three different Numbers, which are exact *Roots* of three *exact Square Numbers*, the two less being exactly equal to the greatest, according to the 47. *Prop. l.* 1. *Euclides*, and the *Sides* or *Roots* making the perfectest sort of right angle triangles, keeping proportion one to another, as 3, 4 and 5. and having constantly these

Angles

Angles *proxime* 90. 53. 8'. and 36. 52'. For example, take the Rod of 4. whose *third, fourth and fifth Cells* contain these three Numbers 12. 16. and 20. the sides of a right Angle triangle and true *Roots* of these three square Numbers 144 256 and 400. Now the two less squares $\frac{144}{256}$ added together, make exactly 400. *the greatest square of the greatest root*. Other right ang. Triang. that have not the said proportion of Sides, and aforesaid Angles, must necessarily have one or more defective Roots for their Sides, which will either come short or overshoot the truth, when we endeavor to square the unsquareable Numbers.

The eighth Point observable is, that though some Columns or Enneads refuse all *Collateral* Addition, (because they have no more Figures in the 8. under cells, than in the Vertical) yet others far more in Number require it. For in the whole following Table of Columns, from 1. to 9999. there are only 127. that refuse collat. addition whereas 9872 require it, in one or more of their under cells. In the single Columns of the 9 Units, only the first or column of 1. refuseth Collat. Add. In the double columns of Tens, only the two first, *viz.* column 10 and 11. In the triple columns of Hundreds, only the twelve first, that is, all from column 100 to 111 *inclusive*, In the Quadruple columns of Thousands, only the first 112 Columns, that is, all from column 1000

to

to Col. 1111 inclusive; All which 127 Columns or bones are void of all collat. add. And therefore all the 8 Under cells in them are marked with Stars, as Signes of non-addition. Note, that no Vertical cells have any collat. add. nor stars before them. Note also, that no *Ennead*, be it never so great, or have many Figures in the Vertical cell, can have any collat. add. in any one Under cell, if the two first Figures of the Vertical begin with 10 or with these three 110. or these four 1110. *&c.* Though all the following Figures be never so great, as 9999 in *infinitum*.

The ninth thing Observable is, that by how much any *Ennead* contains more Figures in its Vertical cell, by so much is it the better to multiply and divide by, since it takes away all *collat. add.* the chief trouble in gathering the products in multiplication, and finding readily the *Quotients* in *Division*. For example, if you turn all the Vertical Units of *Pytha.* Table into one Sum, *viz.* 1 2 3 4 5 6 7 8 9. and multiply it severally by 1. 2. 3. 4. *&c*, it wou'd make an *Ennead* such, as you see exprest in Figure 13. far different from Figure 1. the Table of *Pytha.* whose *collat. add.* it wholy takes away, and yet in substance is the same with the Table.

Concerning the Extensions of Pythagoras *his Table.*

The *Extensions* of the *Pytha.* Table may be distinguished into two sorts, the *greater*, and the *less*. The greater extends it two ways; length way by *Capital Numbers*, and bredth way with as

many *lateral Numbers*: The less extends it only length way by *Capitals* and not by any more Laterals, than are in *Pytha*. Table; which are the 9 Units. For example, the first greater Extension adds to the 9 *Capital* Units of the Table 90 more Capitals: that is, all the whole Numbers of two places between 10 and 99 inclusive: And the like it adds to the 9 Lateral Units, *viz.* 90 more Laterals. As all the cells with their inclosed Numb. in *Pytha*. T. are known to be 81. by multiplying the two greatest Units, 9 Capital and 9 Lateral together; so by multiplying 99 by 99 the two greatest *Capital* and *Lateral Numbers* of two places, you will find the Cells of this first greater Extension to be 9801. The first less Extension adds to the 9 Capital Units (as did the first greater Extension) 90 Numbers of two places from 10 to 99 inclusive, but adds not any one Number to *Pythagoras* his 9 lateral Units. The cells of this Extension by multiplying 99 its greatest Capital by 9, the greatest *Lateral*, are found to be 891 which is not the 10th. part of 9801 the cells of the first greater Extension. A Table of this kind of extent, containing 9801 cells would be very useful, and being of a Moderate largeness, occupying about 10 or 11 pages in *Folio*, might be easily made, as some have heretofore done: Mr. *Joh. Darling* and others. But in this present Table we forbear to place it and all other Extensions of the greater sort, by reason of their Vast largeness and labour in making and using them. In the following Table

the one of 5 *greater Extensions*, the other *of* 5 *less* Extensions, you may see their differences and how many cells, pages and Tomes in *Folio*, each one would contain. But first observe, that we allow a *Folium* to be 14 Inches long, and 8 broad, prescinding from Margents, one page whereof will contain 112 square Inches, In every page reckon 900 cells: In every Tome a 1000 pages.

Five greater Extensions of *Pythagoras* his Table:

Extensions	1 st.	2d.	3d.
Multiplicand	99.	999.	9999.
Multiplier	99	999	9999.
Cells.	9801.	998001.	99980001.
Pages.	10 $\frac{801}{900}$	1108 $\frac{801}{900}$	111088 $\frac{801}{900}$
Tomes.	0.	1 $\frac{109}{1000}$	111 $\frac{089}{1000}$

Extensions.	4 th.	5th.
Multiplicand	99999.	999999.
Multiplier.	99999.	9 9999
Cells.	9999800001	999998000001
Pages.	11110888 $\frac{801}{900}$	1111108888 $\frac{801}{900}$
Tomes.	11110. $\frac{889}{1000}$	111108 $\frac{889}{1000}$

 Five

Five less Extensions.

Extensions	1st.	2d.	3d.	4th.	5th.
Multiplicand	99	999	9999.	99999	999999
Multiplier	9	9.	9.	9.	9
Cells	891	8991.	89991	89999	8999991.
Pages	$9\frac{891}{900}$	$9\frac{891}{900}$	$99\frac{891}{900}$	$999\frac{891}{900}$	$9999\frac{891}{900}$
Tomes	0	0	0	1-	10-

In these Tables, every Extension is exprest by 5. oblong Quadrats, one under another. In the first Quadrat is the Number of the Extension, First, second or third, &c. In the second Quadrat are two Numbers, a *Multiplicand*, and *Multiplier*, being the greatest *capital Numbers*; and the greatest *lateral Number* of that present Extension. In the third Quadrat is the *Product* of the abovesaid *multiplicand* and *multiplier*, or Number of cells of the Extension. In the fourth *Quadrat* is the Number of pages in folio, which that Extension would make. Divide the cells by 900 and the *Quotient* will give the pages. In the fifth Quadrat is the Number of Tomes which that Extension would make. *Divide the Pages* by 1000. and the *Quotient* will give the Tomes. The Extensions both of the greater and lesser sort may be made *in infinitum*, though these two Tables exhibit only five of either sort. It is incredible to our first apprehensions, what a vast

space

ſpace would be taken up by a Table of the fifth greater Extenſion, wherein, as you ſee, 999999. *capital Numbers* are ſuppoſed to be multiplied by ſo many Laterals, and to produce the Number of cells 99999800000001: and conſequently, according to Allowances above-mentioned, pages 1111108888$\frac{301}{900}$, and Tomes in folio 1111108, each Tome having 1000 pages, and (with its cover) 3 inches in thickneſs. Theſe *Tomes*, if they were ſet on end, contiguous one to another in a ſtreight line, they would make a rank of books above 52 Engliſh miles long. Or if all the aforeſaid pages, their Margents cut off, ſhould be laid cloſe one to another on a plain, they would cover more than 30 ſquare Engliſh miles, or 19200 ſquare Acres.

But ſetting aſide all Extenſions of the greater ſort, we will content our ſelves with the third leſs Extenſion, in which as the Table ſhews, 9999. is the *greateſt capital Number* (*Multiplicand and Diviſor*) and 9. the greateſt lateral Number (*Multiplier and Quotient*. The product of cells is 89991. The pages in folio are 99$\frac{821}{900}$ which ſcarce make the 10th. part of a Tome in folio. And obſerve that 9999 contains all the capitals, both of the *Pythagorean Table*, and of the firſt,

 ſecond

second and third less Extensions. For 9 (the Unit on the right hand) counts the 9 Units of the Pythagorean Table; the next 9 counts 90 Numbers of two places, from 10 to 99 exclusive, and makes the first Extension. The third 9. counts 900 num. of 3 places, from 100 to 999. and makes the 2d. Extension: The 4th. 9 counts 9000 num. of 4 places, from 1000 to 9999. and makes the 3d. Extension observe also, that the foresaid numbers, of 990. 900. and 9000. added together make just the number of 9999: and being multiplied severally by 9 do produce severally these num. 81. 810. 8100 81000. all which added together, make up the just num. of cells of the 3d. less Extension *viz.* 8999. Observe lastly, what we touched before speaking of Pytha. Table, that every Capital number from 1 to 9999 being multiplied by all the 9 Units or single figures, produces 9 distinct numbers, one greater than another, which being orderly placed and perpendicularly one under another, make a certain column, whose length is divided into 9 equal parts or cells, the Seats of the 9 Numbers produced, the Capital being the highest. Wherfore there being 9999 Capitals in this present Table, there must be also 9999 Columns, which in substance and in effect are the *Numbring Nines*, *Enneads*, *Rods* or *Bones*, or what else you please to call them: and not only the single Rods of Units (as they were first invented, and hitherto too commonly used) but

double

double Rods of Tens, Triple Rods of Hundreds, and quadruple Rods of Thousands: So that whatever Operation can be performed in matter of Multiplication or Division, by 1. 2. 3. or 4. of the single Rods, the same may be performed by one Rod or column of this Table, and with far greater expedition, without any collateral Addition. For here are actually Tabulated to your Hand all and every whole Number (*Multiplicands and Divisors*) under 10000, and ever with one column or Rod alone. Nay, it will not be hard to work by two columns of this Table at the same time, and then your Multiplicands and Divisors may be any Number under 100000000. But let us proceed to the third point of the Numbring Rods.

Concerning the Numbring Rods or Bones.

These *Arithmetical Rods* (described by most Authors, who have writ of Arithmetick, since they were first found out) own for their first Inventor, the Right Honourable and Learned *John* Lord *Nepeer*, Baron of *Marchiston*, who put forth a Latin Treatise concerning them, Intituled, *Rabdologia*, that is, a Discourse or Treatise of Rods, calling them *Virgulæ* and *Laminæ*: others have or may call them, *Columellæ*, *Tesseræ*, *Enneades*, adding the *Epithets*, *Numerales* or *Arithmeticæ*. To the same Noble Lord, Posterity is obliged for

another

another Rare Invention of *Logarithmes*, both of them aiming at, and attaining the ſame end, which is, to facilitate and perform with greater diſpatch eaſe and certainty the harder parts of Arithmetick, viz. *Multiplication, Diviſion* and *Extraction of Rootes*. Theſe late years paſt *Sir Samuel Morland* moſt ingeniouſly invented two Arithmetical Inſtruments to the purpoſe aboveſaid, for which he deſervs ſingular praiſe. Though the Inſtruments in themſelves be excellent and uſeful, yet they have been hitherto more ſparingly uſed for theſe two reaſons Firſt, becauſe few Artificers are found, who have Hand and Head ſufficient to make them ſo exactly as is requiſite. 2ly, becauſe the Vulgar ſort either wants Heads to comprehend them, or purſes to purchaſe them, being ſomewhat chargeable: whereas the Rods of the Lord *Nepeer* are plain, eaſie and Cheap. He happily fell upon the conceiving and deviſing of them by throughly conſidering the *Pythagorean* Table, in which as before I mentioned, an incredible variety of great and little numbers is found orderly *Tabulated, multiplyed and divided*, with apparent *Multip'yers, Diviſors* and *Quotients*. For example, He ſaid 1234. Tabulated by the firſt 4 capital Columns of the Table, which he multiplyed by 9 and found the *product* in the 9 cell to be (*Collateral additions being obſerved*) 11106. This product he perceived to be alſo a *Dividend*

whoſe

whose *Divisor* appeared 1234. and *Quotient* 9. or *vice versa Divisor* 9. and *Quotient* 1234. Further he Noted that 1234. inverted was 4321. This Number he also found Tabulated together by the first four *Transverse Columnes*, which multiplied by 9. gave 38889 in the 9th. *Capital Column*. This being also a *Dividend* shewed 4321 for *Divisor*, and 9. for *Quotient*, or *viceversa*, &c. But taking a middle Number between 1234. and 4321. For Example, 3142. or 2413. he was at a loss, not finding them Tabulated together, nor the Product lying together, but was to be picked out here and there, not without trouble of the Head and delay of time. This inconvenience hapned as he well perceived, because the Table was always made with its Columns fixt in the same gradual Order of Unites, encreasing from 1. to 9. But the Remedy of this inconvenience soon occurred, which was to unfix the fixt columns by cutting them asunder, and making them moveable, apt to be placed in what order he pleased, as occasion required. Thus were the famous *Numbring Rods* extracted and dissected out of the Pythagorean Table, and in reality are nothing else but the Table it self cut out into its Columns, adding thereunto 3 more for the *square*, *cube* and *cypher*-Rods; such as you see in Fig. 2. It is true, the Lord *Nepeer*, to mind the Operator of *collateral Addition*, drew *Diagonal Lines* through all the 8. undercells of every column or

Rod, whereby frequent Rhombes of this shape appeared in the Rods Tabulated; And whatsoever 2. Figures should be found in one *Rhombe*, they were to be added and made one Number. See Fig. 5. Others by making *a cross Diagonal* in the undercells, included the 2. Figures to be added, in this kind of Diamond form See Fig. 6. Others again included them in a round Circle. See Fig. 7. But because all these three Wayes seem to offend the Eye, and breed Confusion by so many Lines; Others with much less ado note all the added Figures with this mark = or this - declaring that every Figure so Noted requires collateral addition, if Tabulated on the right Hand with another Rod or column on the left. See Fig. 3. yet because the far greater part of the undercells in the Pythagorean Table, (having one Figure more in them then is in the capital) would require this mark = to wit, 58 cells, whereas only 14 cells refuse it: Again, because this mark = and one Figure more then is in the capital require a greater bredth of Rod, I have rather chosen to put an Asterisk *, as a sign of *non-addition* to a few cells and lesser Rods, then this Sign of Addition = to four times more cells and larger Rods, declaring that the Star in any undercell hinders *collateral* • *Addition*; and where the Star is not in an undercell, there must ever be *collateral addition*, if another Rod be Tabulated on the left Hand with it.

But

But here obſerve, that when any (9) hath a Star before it, and (1) carried to it, by reaſon of a Rod Tabulated on the right Hand of it, then that (9) becomes 10, and is capable of lateral addition, if another Rod follow on the left Hand. Obſerve alſo, that all theſe Enneads 1. 11. 111. 1111. 11111. and the like *in infinitum,* require Stars in all their undercells, unleſs when a (9) becomes 10 by (1) carried to it, as now we ſaid. Note alſo, that all leſs Numbers then theſe, having equal places or Figures with them in the Vertical, require Stars in all Undercells: For example, 1111. is an Ennead of four places, and ſo is 1000, a leſs Number yet of four places; ſo is 1001. 1002. 1003. and ſo on till we come to 1110. all leſs Numbers then 1111. but all of four places, and requiring Stars in all their undercells. But whatſoever Number of four places is greater then 1111. as is 1112. 1113. 1114. and ſo on till 9999. then infallibly it will reject the Star, and require lateral addition in one or more of the undercells. See the eighth Obſervable.

Moreover, to avoid Multiplicity of Lines, as much as may be, in the Rods, I reduce 8 of thoſe lines to 2. which formerly ſeparated the 9. cells from one another, as you may ſee in Fig. 8. 9. 10. and 11. For dividing the length of the Rod into three equal parts by two lines, I place the three higheſt cells in the firſt Diviſion, three others in the ſecond, and the three laſt cells in the

last division: See Fig. 8. 9. 10. and 11. according to this Model of placing Stars before certain undercells, (*viz.* such as have equal Number of Figures with their capital cell) and dividing every Rod or column into three equal parts by two Lines; I made a Table, wherein all Capital Numbers from 1. to 99 *inclusive*, were multiplied by the 9. *lateral Units*. which Table being directly the first less Extension of *Pythagoras* his Table, I caused to be cut in brass some years ago, and a few Copies to be printed for my own and other Friends use. At that time I had in prospect the other two less Extensions (2d. and 3d. of *Pyth.* Table) which soon were compleated, and that very readily, by the help of the double Rods (whereof I had made some Sets) and the Table of the 1 less Extension now mentioned: For laying one double Rod at a time to the Columns of that Table, you Tabulate any number from 100 to 9999. and see immediately the *product of multiplication* in all the 9 cells. The other Numbers from 1 to 99 the Table it self Tabulates and multiplies. See a printed Copy of the Table, inserted in *pag.* 27. As the single Rods of my Lord *Neper* were cut out of the *Pyth.* Table, so both single and double Rods have been cut out of the Table of the 1 less Extension, and found by Experience of 9 or 10 years to double the usefulness of the single Rods. For first they sooner Tabulate any great number with fewer Rods. 2dly they Tabulate the self same

ſame number with great variety of Rods, differing in *Specie* one from another. See Fig. 12. 3dly they take away more then the half of collateral Additions, the chief trouble of numbring Rods. 4thly they more readily ſhew the product of multiplication and Quotient of Diviſion in great numbers and fewer Rods. Two of the double Rods reach to any number under (*a*) 10000 three of them to any under (*b*) 1000000. Four of them to any under (*c*) 100000000. &c. This and more the double Rods perform by themſelves. But joyn or Tabulate them with the Table of the 3d Extenſion, and they will moſt readily multiply and divide vaſt numbers. For one Rod and the Table reaches to (*d*) 1000000. Two to (*e*) 100000000. Three to (*f*) 10000000000. &c. To uſe them with the Table of 9999 columns, it is neceſſary, that the Rods be of the ſame length with the columns,

Collat. Additions
(*a*) 1 *at the moſt*
(*b*) 2 *at the moſt*
(*c*) 3 *at the moſt*
(*d*) 1 *at the moſt*
(*e*) 2 *at the moſt*
(*f*) 3 *at the moſt*

though

though the ſame bredth is not preciſely required.

The Rods having on them all the Capital Numbers from 1. to 99 they will require either 50 thin two-faced Tallcys, or 25 ſquare-ſided Parallelopipedons of four faces. It will be convenient to have every Rod twice over, (though once over will be ſufficient if your ſingle Rods of the 9 Units be twice or thrice over,) whereas an ordinary Set of ſingle Bones muſt have every Rod 4. 5. 6. 7. or more times over, according as the Operator deſignes the working of greater or leſſer Numbers.

Another way of ſupplying the want of more Rods of one and the ſame Number, may be by the Table of 9999. Enneads, for in that Table are all Numbers of four places, and conſequently this Number 5757. Beſides, in the double Bones are all Numbers of two places, from 10. to 99. *incluſive*, and conſequently this Number 57. wherefore in the Table and double Bones we have 57. three times over. But ſetting aſide the Table, the Bone alone of 57. is in practice equivalent to three or more Bones of the ſame Number 57. for if you ſet down with your Pen three times 57. thus 575757, as one Vertical Number of one Ennead, you will know what is the Content of every undercell by the undercells of the Rod 57. thrice ſetting them down. For example, the ſecond cell of 57. is 114. which thrice repeated, is 114. 114. 114. Or ob-

ſerving lateral Addition 115514, which is the ſecond cell of the Ennead 575757. In this manner your Operation will be as ready, as if you had had three diſtinct Rods of 57 apiece. There is yet a third way of moſt ready *and clear working, multiplying and dividing vaſt Numbers* of the ſelf ſame Species of Figures, *viz.* all of Nines, or Eights, or Sevens, *&c.* And in what multiplicity you pleaſe, of the ſame Figures, as 3. 4. 5. Nines, yea 10 Nines, 20 Nines, 100 Nines: And ſo of Eights, Sevens, Sixes *&c.* Some 5 ſpecial *Enneads*, or 5 two-faced Rods (or two four-faced ſquare Rods) are required to this ſort of Operation, wherein you will not be troubled, either with any Tabulating of Rods, or collateral Addition. See the Scheme of the ſaid *ſpecial* Rods *pag. ult.* Fig. 14 where obſerve that the nine ſingle Units occupy ſeverally nine Vertical cells, and their undercells to contain for the moſt part only 3 Figures, one leading on the left Hand, another in the middle, a 3d. ending on the Right Hand. Some few undercells (not above 8 in 72) have 4 Figures in them, and then the two laſt on the Right Hand are ending Figures. The middle figure is moſt remarkable, and more then it appears; For in Operation it is to be repeated, or taken ſo often over, as there are Figures of one kind in the ſuppoſed Vertical, abating one: for example, Suppoſe

the *Vertical to be Ten Nines*, or 9999999999. In

2d. Cell of the special Rod (9) are these three figures 198. where (9) in the middle between (1) and (8) is truly nine times nine, that is, one nine less, then Ten nines in the Vertical: So that the

said 2d. cell 198, is in operation 1999999998 or the Vertical multiplyed by (2) This Rule is Universal, yet hath two exceptions; first when any cell hath four figures in it; 2dly. when any cell hath a Star prefixt before it, according to what is abovesaid concerning Asterisks, then infallibly the middle figure is to abate, not only one, but two of the number in the Vertical. One example will clear all. Let ten fours or

4444444444. be given for a Multiplicand, and 279 for a multiplyer, then in your special Rod of 4 Vertical, take out the ninth cell, apparently 396, but really 3999999996. the middle figure (9) requiring to be repeated nine times, or one less, then the number of fours in the Vertical. Next take out the 7th cell, apparenly 3108 but really 3111111108. the middle figure (1) requiring Eight repetitions, or two less then Ten of the Vertical, because this 7th cell hath 4 figures in it. Lastly take out the 2d cell, apparently * 888. but really * 8888888888. because the middle figure (8) requires 8 repetitions (besides the leading and ending 8) or two less then Ten of the Vertical, by reason of the Star

prefix

prefixt before the ſecond cell. The Work ended would appear thus in Multiplication.

Multiplicand	4444444444	39999999996	*cell.*9
Multiplier	———— ——— -279	31111111108	*cell.*7
		8888888888	*cell.*2

Product 1239999999876

In *Diviſion*, *Multiplicand* the *Diviſor*, *Product* the *Dividend*, and *Multiplier* the *Quotient*.

Dividend

Diviſor 4444444444) 1239999999876 (279 *Qu.*
2---8888888888
35111111107
7--31111111108
39999999996
9—-39999999996
00000000000

The ſquare and cube Rod ought to be once over in every Set, with three or four cypher-Rods, as you ſee expreſt in Fig. 11, All are to be ſo orderly placed in a neat pocket caſe, that every Rod be known what Number it hath, by a mark or figure, even before you take it out of the Caſe. If you pleaſe to reprint the Table of the firſt leſs Extenſion by it ſelf in a page, (with its columns, equal to the Columns of the great Table of 9*9.) and give it a Varniſh to laſt the longer, you may immediately thence make Sets of the double bones, meerly by cutting out the capital

E column

columns, and passing them on Talleys of Wood or other matter (two-faced or four-faced) of the same length and bredth with the columns. Besides, as is abovesaid, this little Table of the first less Extension, with one double Rod applied unto it at a time, performs all the whole Work of the great Table of 9999 fixt columns, only with this disadvantage, that it will often have one collateral Addition, whereas the great Table will have none. Notwithstanding the great performances of the less Table, the great Table hath many special uses, for which it deserves to be published, especially not being of any great extent, nor making any great bulk. By advice of Judicious Friends, I thought good to put it forth in a *Duodecimo*, as a convenient *Enchiridion*, or pocket-book, every four pages containing 100 columns with their 900 cells, or every page 25 columns with 225 cells. In which case the Table alone would require 400 pages in *Duodecimo*. But perhaps it will be better, to contract the pages to half the Number, *viz.* to 200. in this manner. Let every page have five Ranks of Columns one under another, each Rank consisting of Ten Columns: so will every page contain 50 Columns, and wheresoever you open the book, the two pages before your eyes, will shew you a just Hundred of Columns. To find the Number you seek for, more readily, you may Tack to the Margent little outstanding Labells, or Indexes, shewing

ing before you open the Book, where every 100 and 1000 begins, such as are seen in certain account-books of Merchants. Let the Book be so bound, that wheresoever you open it, the leaves on either side, may lye Flat without any uprising; For so it will be more easie to Contabulate the Rods with the Table, when occasion requires. Perhaps it would be better to print it in a little Folio, for use at home in your Closet or Library; for then every page would contain its Hundred of Columns, easie to be found out by their own natural order.

As well the forementioned Table of 9999 fixt Columns, as the single and double moveable Rods serve equally in Decimal and common Arithmetick; yea in Decimal, they in a manner take away all the trouble of Division. Neither do they require any particular Rules in operation, different from those which have been delivered concerning the Lord N single bones, by himself in his *Rabdologia*, by *F. Andrew Taquet*, *Sir Jonas Moor*, Mr *William Leybourne* and others in their Arithmetical Treatises. Wherefore I shall say no more of them, but only shew by single examples, (one of multiplication, another of Division, the 3d of extraction of the Square Root, the 4th of the cube Root) the ordinary use of them.

Example of Multiplication.

In *multiplication* commonly it is best to Tabulate the greater Number as *multiplicand* when one

is greater then the other. For example. *Tabulate* 4628 to be *multiplyed* by 72. Place Unit under Unit, as in the Margent (*a*) Then for 2 (the Unit of the *Multiplyer*) take the 2d cell of the *Multiplicand*, *viz.* 9256, and for 7 of the multiplyer take the 7th cell of the *Multiplicand*, *viz.* 32396, and set both cells down, as you see in (*b*) Add the two cells together, and the total Sum or product of multiplication will be 333216 as you find in (*c*) But if you Tabulate 72 to be multiplyed by 4628 the operation will appear as in (*d*).

```
          4628
      (a)   72
      ---------
      (b) 9256
         32396
   (c)  333216
      ---------
            72
          4628
   (d)     576
           144
          432
         288
      ---------
        333216
      ---------
```

In Division the *Divisor is to be Tabulated*; and it much imports, for the speedier dispatch of your operation, that the *leading Rod of the Divisor* be a great Rod or Column of the Table; a quadruple rather then a Triple, a Triple then a double, a double then a single, if the Number of your Divisor will permit,

Ex-

Example of Division

Let 567556 be a *Dividend*, and 4628 the *Divisor*. set them down as in the Margent (*e*) with a Semilune for the Quotient, and Tabulate the Divisor. Then enquire how often can the *Divisor* 4628. be taken out of the first *partial Dividend*, *viz.* 5678. Only once; Therfore put 1 for Quotient in the Semilune, and Subtract 4628 out of 5678, and there will remain 1050 the work standing (if you dash the Figures dispatcht) as in (*f*). Next bring down 5 out of the *Dividend*, and place it on the right Hand of the remainder, as you see in (*g*) separated with a Comma. This done, seek in the cells of the Divisor for 10505 (the next partial Dividend) or for the next less Number then 10505. In the 2d. cell is 9256 the next less. Put 2 in the Quotient, and *Subtract* 9256 out of 10505. and the remainder will be 1249 to which bring down the other 5 of the Dividend, and dash what is dispatcht. Then will the work stand, as in (*h*)

(*e*) 4628) 5678556 (. . . .

~~5678~~558 (1
~~4628~~ (*f*)
1050

(*g*) 1050, 5

~~5678~~~~5~~56 (12.
~~4628~~
~~1050~~, ~~5~~ (*h*)
~~9256~~
1249, 5

In like manner proceed to find the 3d. and 4th. figure of the *Quotient*, and the work finished will appear, as in (*i*). Others dash no figures at all, nor place the Quotient on the Right Hand of the *Dividend*, but set every quote-figure over against the cell of the *Divisor* from whence it was taken. Every remainder they distinguish with a column from the figure brought down out of the *Dividend*, as you may see in (*k*) When in one Number there happen to be two columns, there will be a cypher in the Quotient; when 3 Commaes, then 2 Cyphers in the Quotient. The like Method serves

(*i*) 4628) 5678556 (1227
4628
1050,5
9256
1249,5
9256
3239,6
3239,6
00000

(*k*) 4628) 5678556
1 - 4628
1050,5
2 - 9256
1249,5
2 - 9256
32396
7 - 32396
00000

in *Extraction of the Square Root*, and much *Facilitates* the examining of the work done. If any Number remain after Division ended, it will be the *Numerator* of a Fraction, whose *Denominator* is the *Divisor*. When you turn Integers into a *Radius of Decimals*, *Division* either ceases, or is easily had by cutting off so many Figures (on the Right Hand) from the *product of multiplication* as are in the *Radius*, excepting one.

Example of the Square Roots Extraction

The Extraction of the Square Root is very ready and plain by the Table, or by the double Rods or both of them together. Let the number given, *whose Root you seek*, be 70476025. Put a point under the Unit, and every Altern Figure with a Semilune after the Unit, as you see in (*l*) The 4 points foretell that in the operation there will be 4 partial *Dividends*, and as many Roots. Then seek in the cells of the Square Rod for 70 (the 1 partial Dividend) or the nearer less number to 70. 64 is the nearest less to 70 in the 8th cell. Therefore 8 *is your first Root* to be placed in the Semilune, and 64 is to be Subtracted out of 70. The remainder will be 6 the work appearing as in (*m*). For the finding the 2d. 3d. or any other Square Roots following, observe these Rules: First, bring down

(*l*) 70476025 (. . . .
  . . . .

(*m*) 70476025 (8 . . :
 64
 6

the

the next parttial Dividend and joyn it to the last remainder on the Right Hand.

Secondly, double the *Root or Roots found*, and Tabulate that *double* on the left Hand of the Square Rod or (working by the Table) carry the Square Rod to the double in the Table.

Thirdly, seek for the Number (or next less) of your last remainder joyned to the next *partial Dividend* in the cells of the Tabulated Rods and the *Cell* wherein it is found, will give you the next Root.

Fourthly, Subtract the cells Number out of the remainder and partial Dividend, and proceed as before, wherefore in our present example to find the 2d. Root,

First, bring down 47 and joyn it to 6 to make 647.

Secondly, *Double the Root* 8 and *Tabulate* 16 with the Square Rod.

Thirdly, seek 647 (or the nearer less) in the Tabulated Rods the third cell gives 489, the next less, which Subtracted out of 647, leaves the remainder 158. The 3d. cell gives 3 for the 2d. Root: see the Margent ( ) For the finding of the 3d. and 4th. Root, proceed as before. The

(n) 78476025 (83
64
647
489
158

whole

whole Operation ended stands as underneath at (o) or according to the Method, mentioned in Division, underneath at (p) where any number remains after the work ended, it is the Numerator of a Fraction, whose Denominator is the double of all the Roots and one Unit more. But if you desire a more exact Fraction, add to the Numerator 2. 3. or more Couples of Cyphers, and work as before, and you will find the nearer Decimal Fraction.

(o) 70476025. (8395
64
6,47
489
158,60
15021
839,25
83925
00000

(p) 70476025
8 - 64
6,47
3 - 489
158,60
9 - 15021
839,25
5 -- 839,25
00000

Example of the Cube-Roote's Extraction.

1st. set down the number (whose Cube-root you seek) with a point under the Unit and every 3d.

Figure, and a Semi-Lune for the Roots, as underneath at (q) how many points, so many partial Dividends and Rootes will be in the Operation.

2ly, Seek in the Cube Rod for 94 or the nearest less number: In the 4th cell you find the nearest 64 Set down 4 for the 1 Root and Subtract 64 out of 94, the remainder will be 30; and the work appear as in (r)

For the finding of the 2d, or any other following Root, observe these Rules.

1st. Bring down the next partial Dividend 818 and Joyn it to the remainder 30, on the Right Hand, as in (s)

2ly, Tabulate the triple of Root or Roots found (*Root* 4 *the triple* 12) with Rod or Rods apart call them for distinction, *Right Hand Rods.*

3ly. Tabulate the triple of the Sq. of the Roots found (Root 4 Square 16 the triple of 48) with Rod or Rods, placed on the left Hand, of the Cube Rod: call these left Hand Rods. Or working by the Table, carry your cube-Rod to 48 in your Table.

4ly, Seek for 30818 the present partial dividend or next less number, in the cells of the left Hand Rods. In the 6th cell you find 29016 the next less yet indeed too much, as will after appear. Set this

(q) 93818816 (4..

(r) ~~94~~818816(4..

~~64~~

30

(s) 30,818

this

this number down apart, and draw a line above it, as you see in (t) over the Unit; and above the line place 6 the number of the cell, out of which 29016 was taken: On the left Hand of 6 place the Square of 6, *viz.* 36 as you see in (u) then take the 6th and 3d. cell (by reason of 36 the Square) out of the right Hand Rods, *viz.* 72 and 36 and place them as you see in (w) adding all the numbers under the line into one Sum, *viz.* 33336, as you see in (x) which being too great to be taken out of 30818. you must go back and take a less cell, then 6.

(t) 39.6
29016
(u) 366
29016
(w) 72
36
(x) 33336
(y) 255
24125

Take therefore the cell 5. which hath in it the the number 24125. write it apart with a line above it, and an above line over the Unit: place 5 (the cells number) and on the left Hand of 5 the the Square of 5 *viz.* 25 as in the margent (y) take out of the right Hand Rods the 5th. and 2d. cell (by reason of the Square 25) *viz.* 60 and 24 and add them, as you see in (z) to make 27125 which taken out of 30818 there will remain 3693 and the work stand as in the margent

(z) - - 25 , 5
24125
60
24
27125

(a) ~~9~~48188~~1~~6 (45
~~6~~4
3~~0~~,8~~1~~8
~~27~~1~~2~~5
3693

gent. (a) For the 3d Root, do as you did for the 2d.

1st, Bring down the next Dividend and joyn it to this last remainder.

2ly, Tabulate a part on the right Hand Rods, the triple of the Roots found.

3ly, Tabulate the triple of the Square of the Roots on the left Hand of the Cube-Rod.

4ly, Seek in the cells of the left Hand rods for the left Hand rods for the partial Dividend.

5ly, Set down apart the number required *found*, and draw a line above it: above that line and over the Unit place the Figure of the cell taken, and on the lefthand of the figure, place its square, as was expressed as before above in the margent, (a) the whole Operation ended, will appear briefly as in (b)

(b) 948 18816. (456
64
30,818
27125
3693816
3693816
0000000

Note first, that scarce can you give any precept in writing concerning Extraction of rootes, so clear, but that they shall confound or puzzle a young Student of Arithmetick, who will be able to learn more in an Hour of a Master shewing him

him the practice, than in a day or week by his own reading of precepts.

2ly, note, that in Cubick Extractions it is not easie to foresee or prevent the taking of too great a number out of the *left Hand and Cube-Rods*. We may probably conjecture that it will happen so, when the number taken is almost as great as the partial Dividend, and yet is to be increased by adding 1 or 2 cells more out of the *Right Hand Rods*.

3ly, Note, that when the capital cell of the left hand and cube-rods is greater than the partial Dividend, a *Cypher* is to be put in the *Quotient* as a *Root*, and the next partial dividend is to be brought down and joyned to the former.

4ly, note, that if any number remain after Extraction, it must be set down as the Numerator of a common Fraction, whose Denominator is a number made of the *triple of all the* rootes, and of the triple of the Square of all the rootes, and an Unity. For example. The rootes being 456 the *Denominator* would be 625177 See the Margent (c) But far better it is to add to the Numerator, or the remaining number, 3 or 4 triples of cyphers thus, 000, 000, 000. and work out by the precedent rules a clear and plain decimal Fraction.

(c) *Triple of Rootes.*	1368
Triple of Square.	623808
Unity ————————	1
Summa	625177

Thus much (and indeed more than I first intended) concerning *Pytkagoras* his Table, the Extensions thereof, and the Numbring Rods.

And

And here I might (had I not been too long already) exemplifie in a few instances; and thereby shew, that whatsoever is performed by *Logarithmes* in Problemes of Trigonometry, *Sines*, *Tangents*, *Secants*, *Questions of Interest*, &c. may be also performed by this Table of the *third less Extension*, and the *double Rods*, or by the double Rods alone: whether with more readiness and clearness, Practice and Experience must shew.

What also can be performed by Mr. *Brigges*'s Table of 20 Chiliades of Logarithmes, may be done (if I mistake not) more plainly and speedily by this Table. For though it be but the half of 20 Chiliades, (it being only 10 Chiliades), yet by applying one double Rod thereunto, it exceeds 20 Chiliades by 980000 Chiliades.

For Conclusion, I will here suggest certain Lines divided into certain digits, which are singularly useful in measuring most things measureable, and make your Operation quick and plain, without trouble of division, or necessity of reducing inches into other known Integers. For though you measure by digits only, and multiply them by one another, yet the Product of Multiplication immediately gives you in hundreds or thousands, the superficial or solid content, not only in digits, but in other known termes of Feet, Yards, Acres, Gallons, Barrels, Bushels, &c. For Wine-Gallon-digits proceed thus; Take the *Cube Root* of 231, (the solid inches in

a Wine-Gallon) which is 6,136 *vulg. Inc. proxime.* *Divide* this Root into ten equal parts exactly, with subdivisions of each part into other ten less parts, and you have the Wine-Gallon-digits.

You measure by them, for example, a cylindrical capacity, and find the Diameter of it to be 56 digits, and height 60 digits, *the area* of that circle will be 24,64 superficial digits, which multiplied by 60 digits, produces 147. 840 solid digits, whereof every thousand is a just Wine-Gallon. There are therefore 147 Gallons, and 840 digits towards another thousand or Gallon: that is above three quarts.

For Beer or Ale Gallon digits, take the cube Root of 282 solid vulg. inches in the Ale Gallons, which is 6,558 *proxime.* *Divide* this Root into ten equal parts, with subdivisions, as above.

For *Beer-barrel-digits,* take the cube Root of 10152 (solid vulg. inc. in a Beer-barrel of 36 Gall.) which is 21,653 *proxime,* to be divided into ten equal parts with subdivisions, as before.

For Foot-digits take the square-Root of 144 (a Foot square) or cube Root of 1728, (a Foot solid) both which is 12 common inc. Divide this Root 12 into ten equal parts, with subdivisions; measure and work by them, every hundred square will be a true Foot square, equal to 144 common inc. and every thousand solid, a Foot solid equal to 1728 com. inc. For example, you measure a tetragon pyramide, whose one square side is 50 Foot-digits, and height 60. The square

of

of 50 is 2500, and gives the area of the pyramide at the bottom, *viz.* 2500 sq. dig. or 25 sq. Feet: multiply the area 2500 by 20 (a 3d of the height 60) and the product will be 50000 sol. dig. that is 50 solid Feet, equal to 86400 solid common inc.

These 4 lines of Wine-gall. dig. Beer-gall. dig. and foot-digits, are of excellent use in Gauging, and measuring any thing by feet sq. or solid, and may be conveniently cut on a Ruler, or long measuring staff, hard by or on both sides of a line of common inches, so that by meer inspection you may see how much they differ amongst themselves, and from common inches. If you desire yard-dig. to measure by sq. or cube-yards, divide 36 the square Root of 1296 (a sq. yard in common inches) and cube Root of 46656 (a yard sol. in common inches) into ten equal parts, as in other digits above; So may you have bushel-dig. by dividing 12,958, which is *proxime* the cube Root of 2176 (commonly esteemed a solid bushel in vulgar inches) into 10 equal parts. For measuring of Land, Mr. *Gunter*'s Chain (of 100 links, equal to 4 perches or 66 foot in length, is very convenient. Every 10000 sq. links is a chain sq. or 16 perch. sq. or the 10th. part of an Acre s. 100000 of links sq. is 10 chains sq. or 160 perches sq. or an Acre sq. Note 1. That when the Root is great, as 20, 30, or more vulg. inc. then you may divide it into more than 10 equal parts, as 100. 1000, &c. Note 2. That in working by the aforesaid Root-dig, the contingent Fractions will be decimal and clear.

FINIS.

ERRATA.

Fol. 32. lin. 12. set them thus 72.

a	4628
	72
b	9259
	32396
c	333216

Fol. 33 l. 21. read thus e 4628) 5678556 (1227

f	4628
g	1050, 5
	9256
h	1249, 5
	9256
	3239, 6
	32396
	00000

Fol. 39 l. 12.

v.	30, 6
	29016
w	72
	36
x	33336

TETRASTICHON.

In Enneadas Arithmeticas.

Conditor Æneidos peperit sibi nobile Nomen;
Nullum Nomen habet Conditor Enneados.
Si tamen Enneados, quæratur, quis fuit Author?
Baro, Refer, Neperus, Pythagorasque fuit.

The Æneid's Author is a Man much Fam'd,
The Ennead's Author not so much as Nam'd;
But if you are askt who th' Ennead's Author was;
Say Lord *John Neper*, and *Pythagoras*.

DISTICHON.

In Tabulam 10000 Enneadum.

Hac Tabula Enneadas decies tibi Mille Ministr
Pythagoræ tantum prisca Tabella decem.

This Table gives Ten Thousand Enneades, when
Pythagora's Old Table gives but Ten.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	1
*2	*4	*6	*8	10	12	14	16	18	*20	*22	*24	*26	*28	*30	*3
*3	*6	*9	12	15	18	21	24	27	*30	*33	*36	*39	*42	*45	*4
*4	*8	12	16	20	24	28	32	36	*40	*44	*48	*52	*56	*60	*
*5	10	15	20	25	30	35	40	45	*50	*55	*60	*65	*70	*75	*8
*6	12	18	24	30	36	42	48	54	*60	*66	*72	*78	*84	*90	*9
*7	14	21	28	35	42	49	56	63	*70	*77	*84	*91	*98	105	1
*8	16	24	32	40	48	56	64	72	*80	*88	*96	104	112	120	1
*9	18	27	36	45	54	63	72	81	*90	*99	108	117	126	135	14
34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	4
*68	*70	*72	*74	*76	*78	*80	*82	*84	*86	*88	*90	*92	*94	*96	*9
102	105	108	111	114	117	120	123	126	129	132	135	138	141	144	14
136	140	144	148	152	156	160	164	168	172	176	180	184	188	192	19
170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	2
204	210	216	222	228	234	240	246	252	258	264	270	276	282	288	29
238	245	252	259	266	273	280	287	294	301	308	315	322	329	336	34
272	280	288	296	304	312	320	328	336	344	352	360	368	376	384	39
306	315	324	333	342	351	360	369	378	387	396	405	414	423	432	4
67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	8
134	136	138	140	142	144	146	148	150	152	154	156	158	160	162	16
201	204	207	210	213	216	219	222	225	228	231	234	237	240	243	24
268	272	276	280	284	288	292	296	300	304	308	312	316	320	324	32
335	340	345	350	355	360	365	370	375	380	385	390	395	400	405	41
402	408	414	420	426	432	438	444	450	456	462	468	474	480	486	49
469	476	483	490	497	504	511	518	525	532	539	546	553	560	567	57
536	544	552	560	568	576	584	592	600	608	616	624	632	640	648	65
603	612	621	630	639	648	657	666	675	684	693	702	711	720	729	73

5	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
0	*32	*34	*36	*38	*40	*42	*44	*46	*48	*50	52	*54	*56	*58	*60	*62	*64	*66
45	*48	*51	*54	*57	*60	*63	*66	*69	*72	*75	*78	*81	*84	*87	*90	*93	*96	*99
60	*64	*68	*72	*76	*80	*84	*88	*92	*96	100	104	108	112	116	120	124	128	132
75	*80	*85	*90	*95	100	105	110	115	120	125	130	135	140	145	150	155	160	165
90	*96	102	108	114	120	126	132	138	144	150	156	162	168	174	180	186	192	198
05	112	119	126	133	140	147	154	161	168	175	182	189	196	203	210	217	224	231
0	128	136	144	152	160	168	176	184	192	200	208	216	224	232	240	248	256	264
35	144	153	162	171	180	189	198	207	216	225	234	243	252	261	270	279	288	297
8	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66
96	*98	100	102	104	106	108	110	112	114	116	118	120	122	124	126	128	130	132
44	147	150	153	156	159	162	165	168	171	174	177	180	183	186	189	192	195	198
92	196	200	204	208	212	216	220	224	228	232	236	240	244	248	252	256	260	264
40	245	250	255	260	265	270	275	280	285	290	295	300	305	310	315	320	325	330
88	294	300	306	312	318	324	330	336	342	348	354	360	366	372	378	384	390	396
36	343	350	357	364	371	378	385	392	399	406	413	420	427	434	441	448	455	462
84	392	400	408	416	424	432	440	448	456	464	472	480	488	496	504	512	520	528
32	441	450	459	468	477	486	495	504	513	522	531	540	549	558	567	576	585	594
1	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
62	164	166	168	170	172	174	176	178	180	182	184	186	188	190	192	194	196	198
43	246	249	252	255	258	261	264	267	270	273	276	279	282	285	288	291	294	297
24	328	332	336	340	344	348	352	356	360	364	368	372	376	380	384	388	392	396
05	410	415	420	425	430	435	440	445	450	455	460	465	470	475	480	485	490	495
86	492	498	504	510	516	522	528	534	540	546	552	558	564	570	576	582	588	594
67	574	581	588	595	602	609	616	623	630	637	644	651	658	665	672	679	686	693
48	656	664	672	680	688	696	704	712	720	728	736	744	752	760	768	776	784	792
29	738	747	756	765	774	783	792	801	810	819	828	837	846	855	864	873	882	891

A **Figure. 1**

0	1	2	3	4	5	6	7	8	9
0	2	4	6	8	10	12	14	16	18
0	3	6	9	12	15	18	21	24	27
0	4	8	12	16	20	24	28	32	36
0	5	10	15	20	25	30	35	40	45
0	6	12	18	24	30	36	42	48	54
0	7	14	21	28	35	42	49	56	63
0	8	16	24	32	40	48	56	64	72
0	9	18	27	36	45	54	63	72	81

B **Fig. 2**

S.1	C.01	0
4	08	0
9	27	0
16	64	0
25	125	0
36	216	0
49	343	0
64	512	0
81	729	0

Fig. 5

1.	2.	3.	4.
2	4	6	8
3	6	9	1/2
4	8	1/2	1/6
5	1/0	1/5	2/0
6	1/2	1/8	2/4
7	1/4	2/1	2/8
8	1/6	2/4	3/2
9	1/8	2/7	3/6

Fig. 6

1.	2.
2	4
3	6
4	8
5	1 0
6	1 2
7	1 4
8	1 6
9	1 8

Fig. 7

3.	4.
6	8
9	1 2
1 2	1 6
1 5	2 0
1 8	2 4
2 1	2 8
2 4	3 2
2 7	3 6

Fig. 8

12	34
*24	*68
*36	102
*48	136
*60	170
*72	204
*84	238
*96	272
108	306

Fig. 13

1 2 3 4 5 6 7 8 9
*2 4 6 9 1 3 5 7 8
*3 7 0 3 7 0 3 0 7
*4 9 3 8 2 7 1 5 6
*6 1 7 2 8, 3 9 4 5
*7 4 0 7 4 0 7 3 4
*8 6 4 1 9 7 5 2 3
*9 8 7 0 5 4 3 1 2
1 1 1 1 1 1 1 0 1.

Fig. 14

1	2	3	4	5	6	7	8
*222	*444	*666	888	110	132	154	176
*333	*666	999	132	165	198	231	264
*444	*888	132	170	220	204	3108	352
*555	110	165	220	275	330	385	440
*008	132	108	204	330	396	402	5328
*777	154	231	3108	385	462	5439	6216
*888	176	264	352	440	5328	0210	7104
*999	198	297	396	495	594	693	792

0
0
0
0
0
0
0
0
0

Fig. 3

A B

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

Fig. 4

S.1	C.01	0
4	08	0
9	27	0
16	64	0
25	125	0
36	216	0
49	343	0
64	512	0
81	729	0

. 8

34
*68
102
136
170
204
238
272
306

Fig. 9

123
*246
*369
*492
*615
*738
*861
*984
1107

Fig. 10

i 2 3 4
*2468
*3702
*4936
*6170
*7404
*8638
*0872
11106

Fig. 11

S. 1	C.01	0	00	000
* 4	*08	* 0	*00	*000
* 9	*27	* 0	*00	*000
16	*64	* 0	*00	*000
25	125	* 0	*00	*000
36	216	* 0	*00	*000
49	343	* 0	*00	*000
64	512	* 0	*00	*000
81	729	* 0	*00	*000

Fig. 12

78,96,78
78,96,7,8
78,9,67,8
7,8,9,678
7,8,96,78
7,8,9,6,78
54,72,54 72
54,72,5,4,7,2
54,7,25,4,72
5,47,25,4,72
5,4,7,2,54,72

	8	9
4	176	198
1	264	297
08	352	300
5	440	405
2	5328	504
39	6216	693
10	7104	792
3	792	891

Fig. 15

Sq.1	C 01	0	*00
* 4	* 08	* 0	*00
* 9	*27	* 0	*00
16	*64	* 0	*00
25	125	* 0	*00
36	216	* 0	*00
49	343	* 0	*00
64	512	* 0	*00
81	729	* 0	*00

4. 9. 16. 25. 36. 49. 64. 81. 10

8. 27. 64. 125. 216. 343. 512. 729. 1000

16. 81. 156. 625. 1296. 2401. 4096. 6561. 10000